Reconstruction method for UMD

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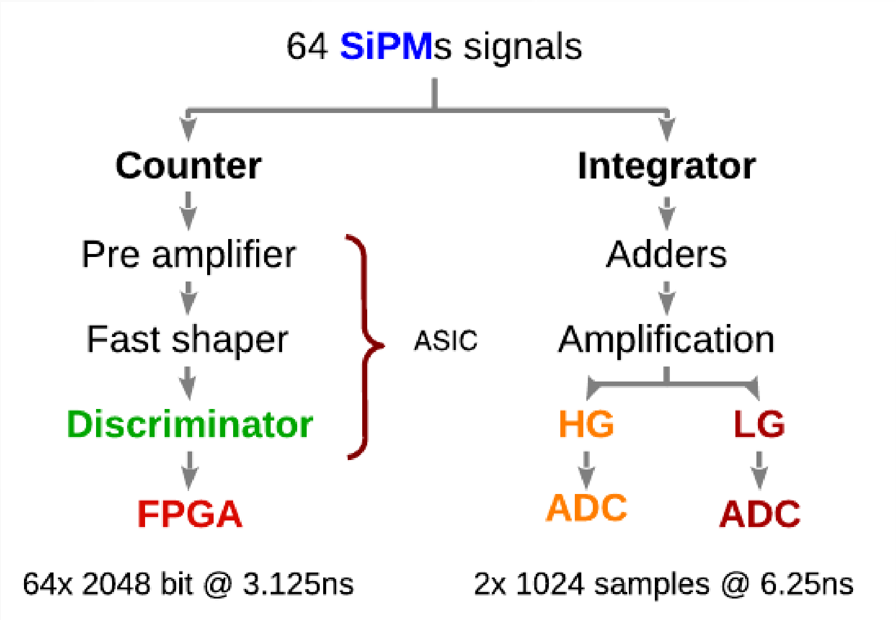
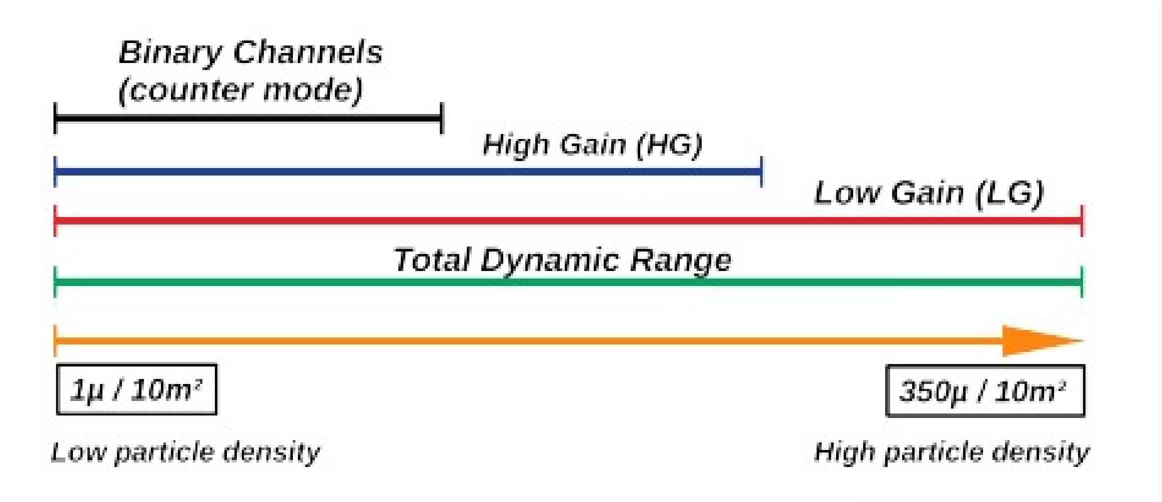
Introduction





Ref 1

[GAP2020_003]

- The integrator will improve the determination of the muon densities closer to the shower core

- Current reconstruction method used for UMD → profile/integrated likelihood method with the detector timing in the counter mode

$$\mu(r) = \mu_0 \frac{g(r)}{g(r_0)}$$

$$g(r) = \left(\frac{r}{r_1}\right)^{-\alpha} \left(1 + \frac{r}{r_1}\right)^{-\beta} \left(1 + \left(\frac{r}{10r_1}\right)^2\right)^{-\gamma}$$

- Likelihood $$L = \prod_{i=1}^{N_{sat}} P_1 \times \prod_{i=1}^{N_{good}} P_2 \times \prod_{i=1}^{N_{silent}} P_3$$

See Ref 5 for more info

Motivation

- To find a likelihood for the reconstruction of the MLDF which includes,
  - Saturated stations
  - Silent stations
  - Stations with small number of muons
  - Stations with large number of muons

by using both integrator and counter modes.

Characterizing AMIGA integrator output with a log-normal function

- Integrator output signal follows linearity.
- Total charge S_n is the arithmetic sum of **n muons**.

$$S_n = \sum_{i=1}^{n} q_i$$

q → charge of one muon following a distribution P(q).

- The estimator of number of the muons $\hat{n}$ is,

$$\hat{n} = \frac{S_n}{\langle q \rangle}$$

→ Mean value of the charge corresponding to one muon

ZTP Distribution

- The number of muons n, that hit a given MD is considered to follow a conditional zero truncated Poisson distribution,

$$P_0(n|\mu) = \frac{\mu^n}{(\exp(\mu) - 1)\ n!}$$

$$\mu = \rho_\mu A \cos\theta$$

$$\langle n \rangle = \frac{\mu}{1 - \exp(-\mu)}$$

$$Var[n] = \frac{\mu}{1 - \exp(-\mu)} - \frac{\mu^2 \exp(-\mu)}{(1 - \exp(-\mu))^2}$$

- n = 0 can be identified with probability one.

- Without assuming a particular shape for P(q) the following expressions are obtained,

$$\langle \hat{n} \rangle = \langle n \rangle$$

$$Var[\hat{n}] = \frac{\mu}{1 - \exp(-\mu)}(\epsilon^2[q] + 1) - \frac{\mu^2 \exp(-\mu)}{(1 - \exp(-\mu))^2}$$

$$\epsilon[q] = \frac{\sigma[q]}{\langle q \rangle}$$

- For large number of muons, $\langle \hat{n} \rangle = \langle n \rangle \cong \mu$

$$Var[\hat{n}] \cong \mu \times (\epsilon^2[q] + 1)$$

$$\epsilon[\hat{n}] \cong \sqrt{\frac{\epsilon^2[q] + 1}{\mu}}$$

} Larger than Poisson

Charge distribution

- The charge distribution of one muon is assumed to follow a log-normal distribution [GAP2020_003],

$$P_{LN}(q) = \frac{1}{\sqrt{2\pi}\,\theta q} \exp\left[-\frac{(\ln q - m)^2}{2\theta^2}\right]$$

$$\langle q \rangle = \exp\left(m + \frac{\theta^2}{2}\right)$$

$$Var[q] = \exp(\theta^2 - 1)\exp(2m + \theta^2)$$

$$\epsilon[q] = \sqrt{\exp(\theta^2) - 1}$$

$$\theta = \sqrt{\ln(\epsilon^2[q] + 1)}$$

$$m = \ln\left[\frac{\langle q \rangle}{\sqrt{\epsilon^2[q] + 1}}\right]$$

- When $\mathrm{Var[q]} \lesssim 1$, the sum of n log-normal random variables is also a log-normal distribution irrespective of the value of n.

$$\boxed{P(S_n) \cong LN(m_n, \theta_n^2)}$$

$$\langle S_n \rangle = n \times \langle q \rangle$$

$$Var[S_n] = n \times Var[q]$$

$$\epsilon_n^2 = \frac{\epsilon^2[q]}{n}$$

$$P(S_n) = \frac{1}{\theta_n S_n \sqrt{2\pi}} \exp\left(-\frac{(\ln S_n - m_n)^2}{2\theta_n^2}\right)$$

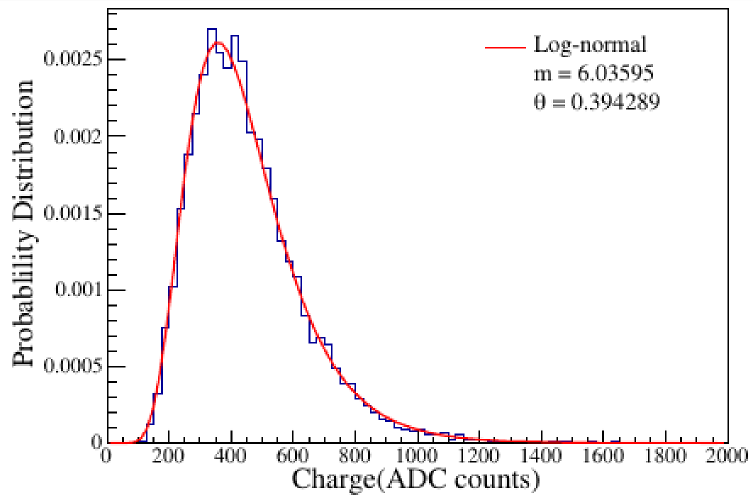
$$\theta_n = \sqrt{\ln\left(1 + \frac{\exp(\theta^2) - 1}{n}\right)}$$

$$m_n = m + \frac{\theta^2}{2} + \ln\left(\frac{n}{\sqrt{1 + \frac{\exp(\theta^2) - 1}{n}}}\right)$$

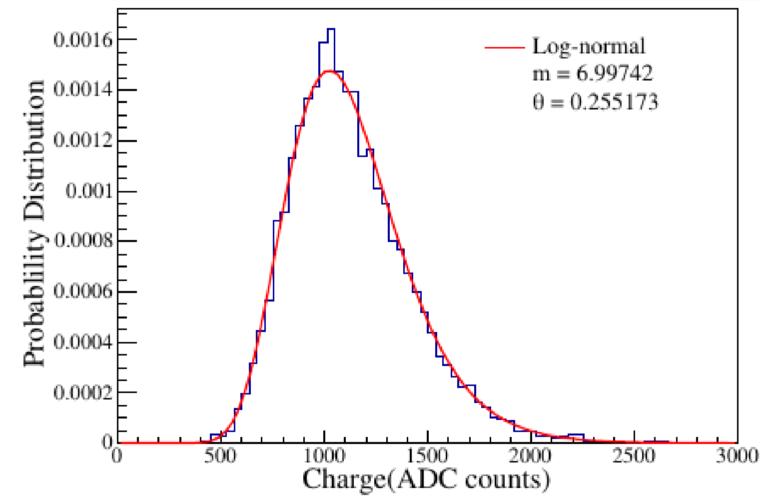
- The log-normal character of S_n is tested from simulations by sampling the distribution n times

$$P_{LN}(q) \longrightarrow \{q_1, ..., q_n\}$$

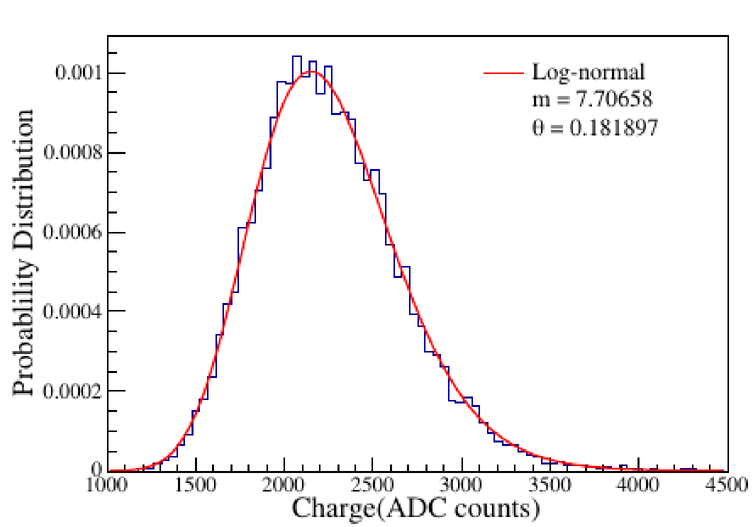
- From integrator calibration data of module ID 108 [GAP2020_003], the equations can be verified
- For LG channel,



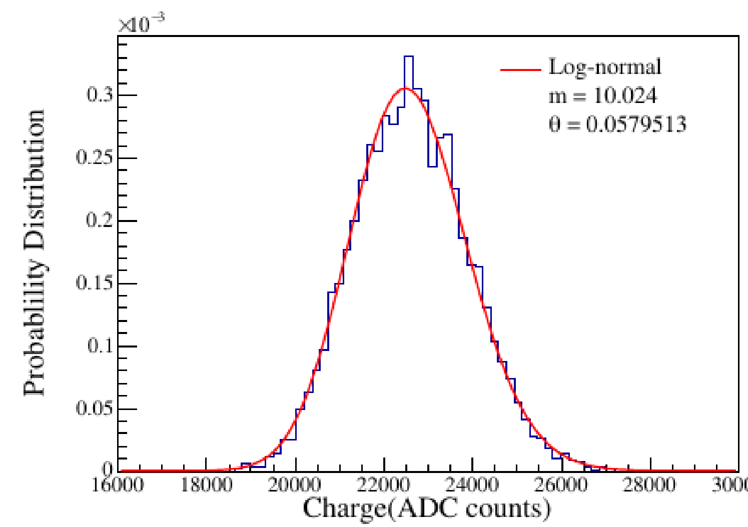
(a) charge distribution for n=2



(b) charge distribution for n=5



(c) charge distribution for n=10



(d) charge distribution for n=100

The distribution function of the number of muons estimator

- In the linear region of the integrator, substitute $S_n \rightarrow \hat{n}$ <q> in,

$$P(S_n) = \frac{1}{\theta_n S_n \sqrt{2\pi}} \exp\left(-\frac{(\ln S_n - m_n)^2}{2\theta_n^2}\right)$$

- The compound probability distribution becomes,

$$P(\hat{n}|\mu) = \sum_{n=1}^{\infty} \frac{1}{\theta_n \hat{n} \sqrt{2\pi}} \exp\left(-\frac{[\ln(\hat{n}) - \ln(\exp(m_n)/\langle q \rangle)]^2}{2\theta_n^2}\right) \frac{\exp(-\mu)\mu^n}{(1-\exp(-\mu))n!}$$

- It is not possible to obtain an analytic expression for the distribution function.

Gaussian distribution

- In the region of high particle density it can be approximated by a Gaussian distribution function.

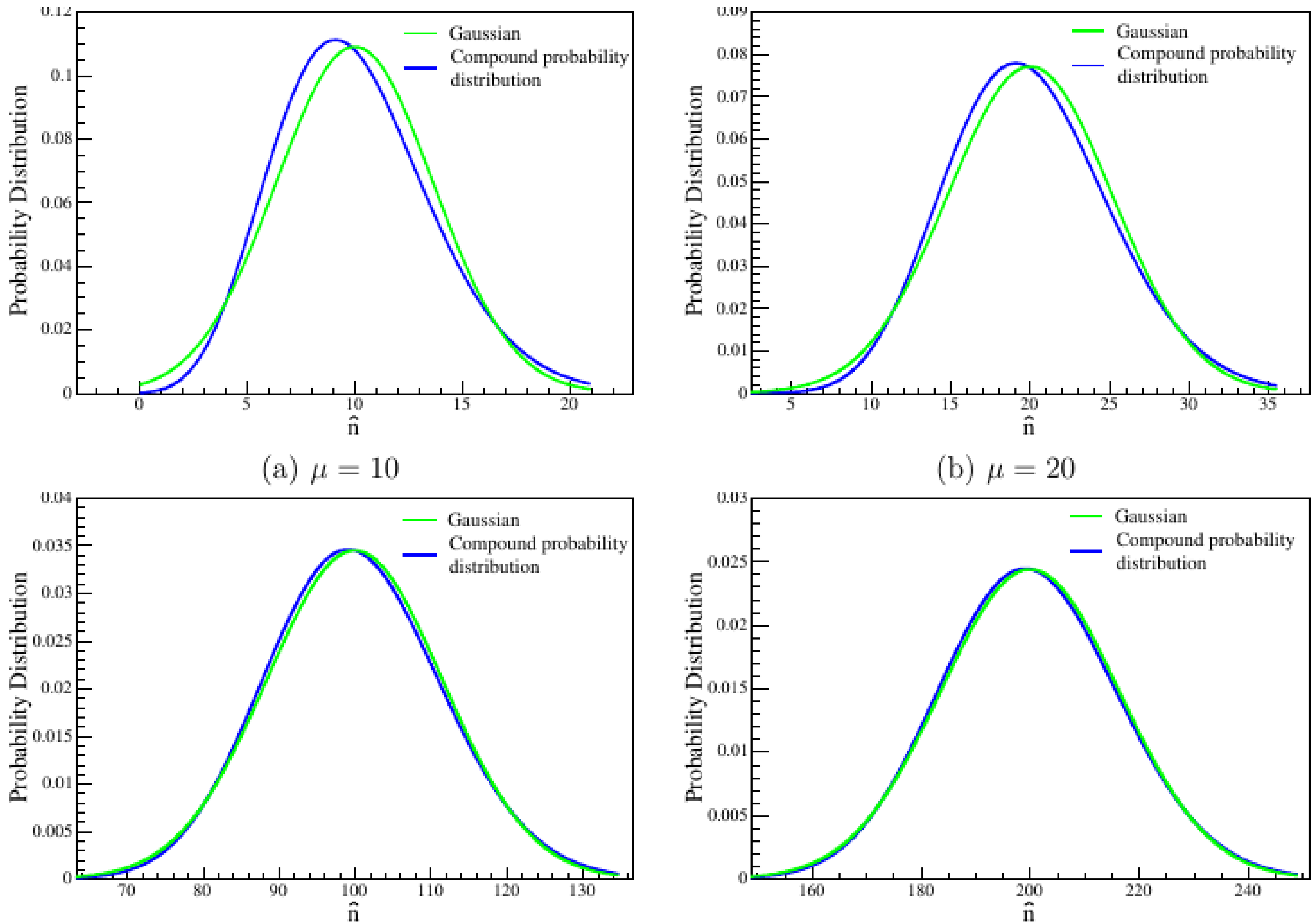
$$P_G(\hat{n}|\mu) = \frac{1}{\sigma[\hat{n}]\sqrt{2\pi}} \exp\left(-\frac{(\hat{n} - \langle\hat{n}\rangle)^2}{2\sigma^2[\hat{n}]}\right)$$

$$\langle\hat{n}\rangle = \langle n\rangle \cong \mu$$

$$\sigma[\hat{n}] \cong \sqrt{\mu(\epsilon^2[q] + 1)}$$

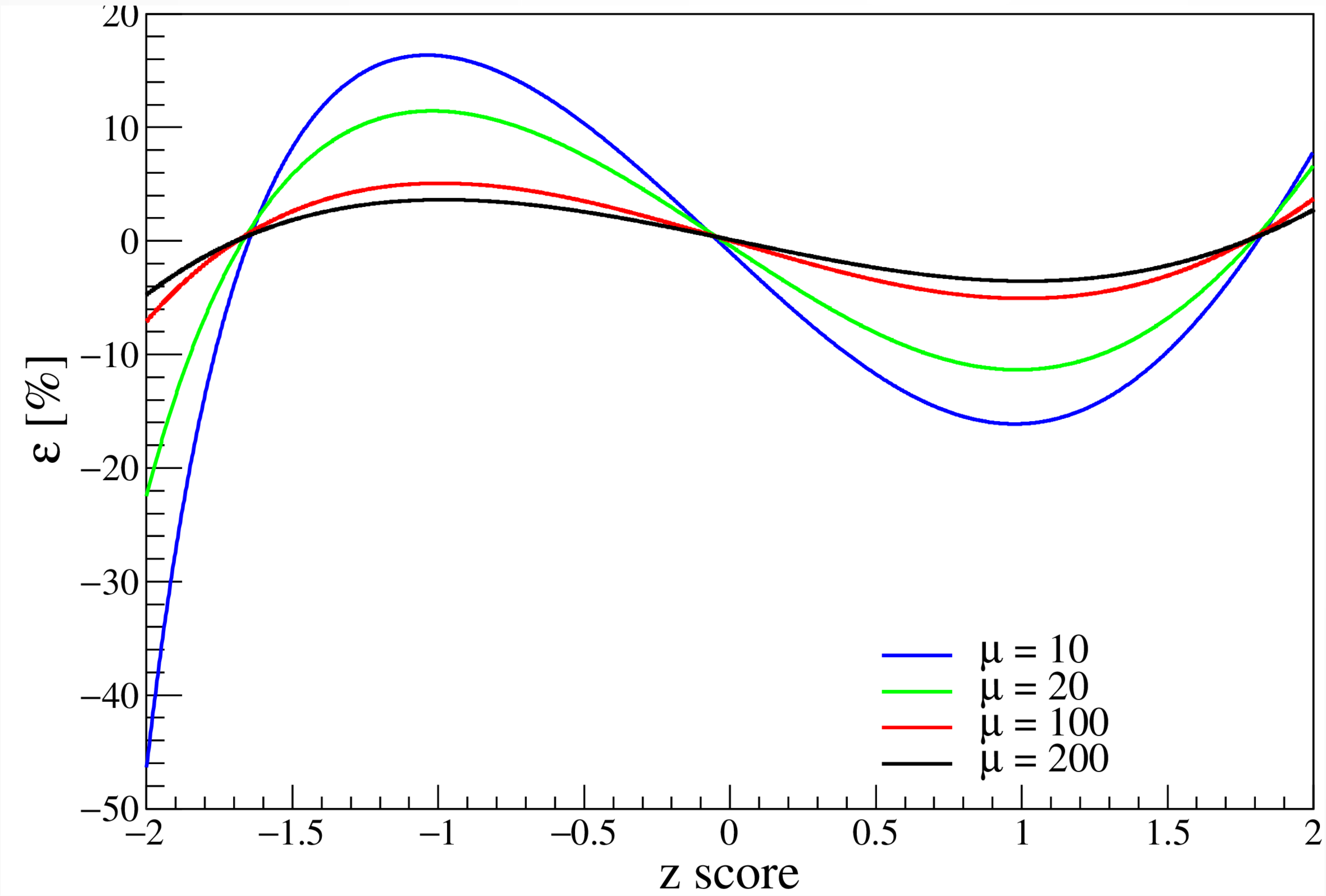
- When it is safe to use the Gaussian approximation?

Comparison of $P(\hat{n}|\mu)$ & P_G in 2σ region of LG channel



(a) $\mu = 10$ (b) $\mu = 20$ (c) $\mu = 100$ (d) $\mu = 200$

Comparison of $P(\hat{n}|\mu)$ & P_G in 2σ region of LG channel



$$z \text{ score} = \frac{\hat{n} - \langle \hat{n} \rangle}{\sigma[\hat{n}]}$$

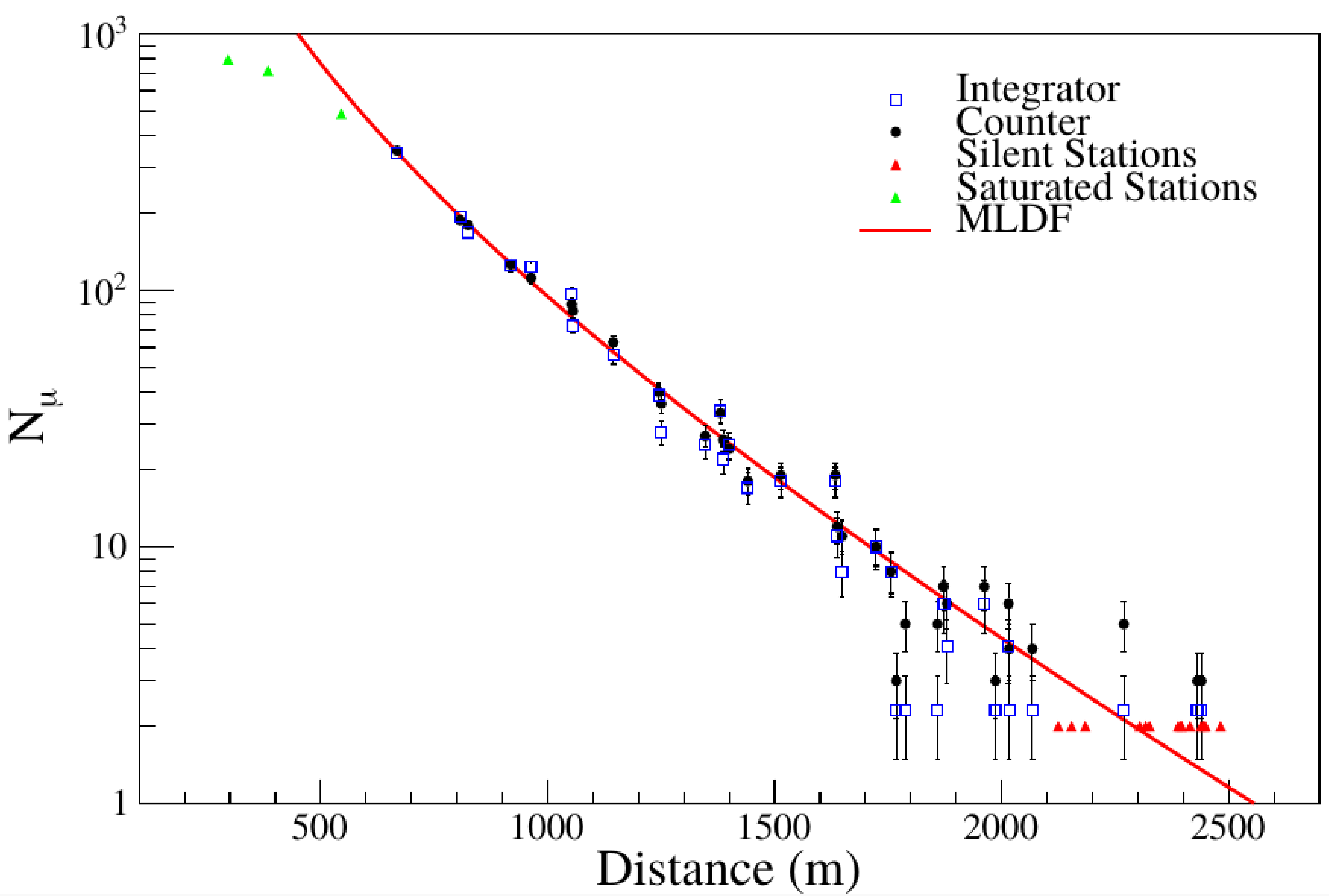
$$\varepsilon = 100\% \left(1 - \frac{P_G}{P}\right)$$

- Note that for a single 10 m^2 module with muons arriving in the same time bin,
  - LG saturates at ~362 muons
  - HG saturates at ~85 muons

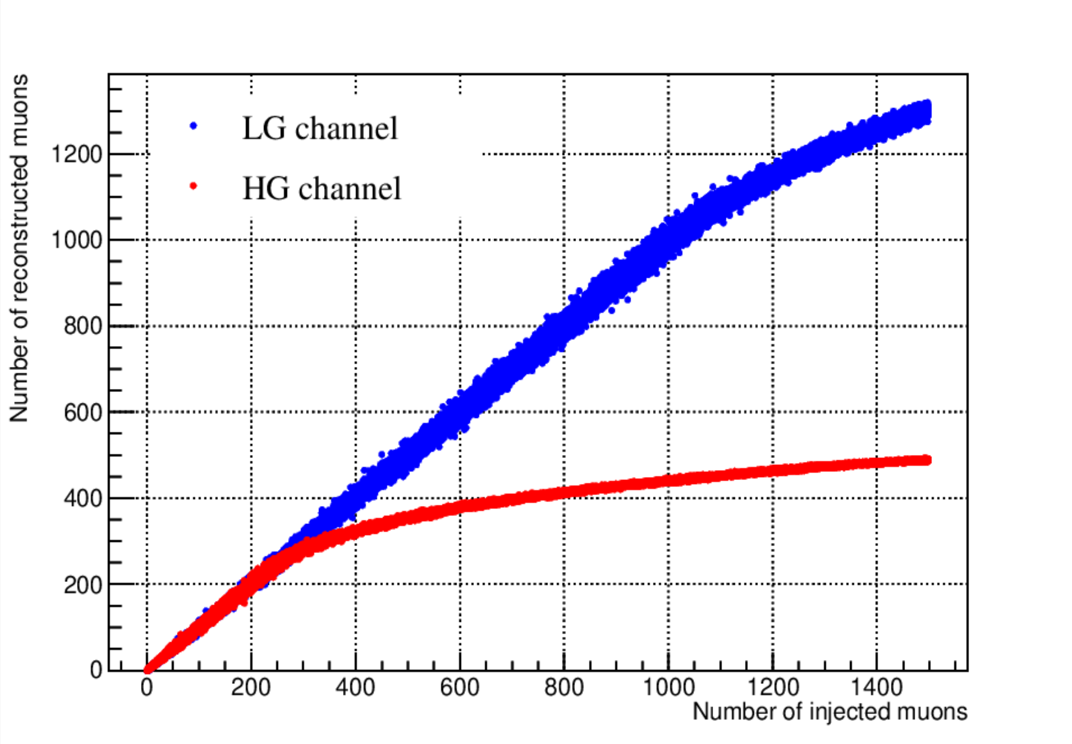
- In the real life scenario where muon arrival is spread over several time bins. For a station,
  - LG saturates at > (**3×362**)
  - HG saturates at > (**3×85**)

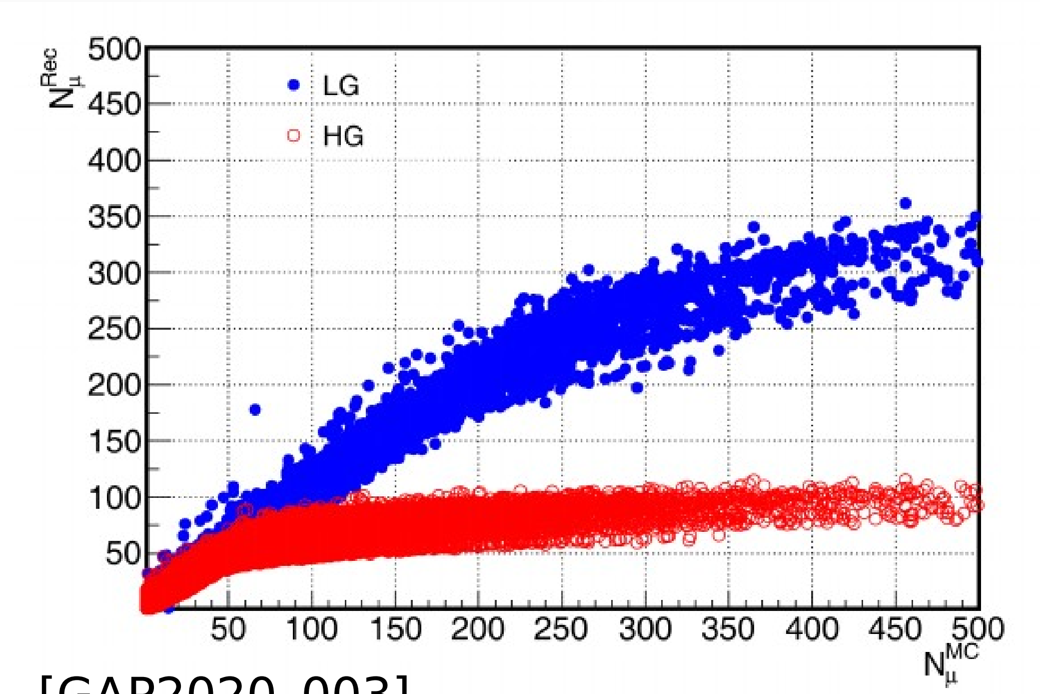
- $\mu=200$ is a good approximation for Gaussian.

Muons as a function of the distance to the shower axis for one simulated event



Primary → Fe
Θ → 35°
E→ 1 EeV





[GAP2020_003]

- Reproduced the plot of number of reconstructed muons as a function of number of injected muons for both channels of the integrator.
- HG channel changes slope at ~260 injected muons
- LG channel changes slope at ~1100 injected muons

Summary and conclusions

- The distribution function for the muon estimator based on the integrator information is obtained.
- It can be seen from the plot that at $\mu=200$, ε is less than 5%. Hence the compound distribution function can be approximated to a Gaussian.
- The same study is done for the HG channel and similar results are obtained.

Current & future work

- Implementation of saturation in integrator simulation
- Developing a reconstruction method including both modes of UMD

References

1) The Pierre Auger collaboration, Design, upgrade and calibration of the silicon photomultiplier front-end for the AMIGA detector at the Pierre Auger Observatory.

2) M. Romeo, V. Da Costa, and F. Bardou, Broad distribution effects in sums of log-normal random variables, arXiv:physics/0211065v2

3) A.M Botti, Determination of the chemical composition of cosmic rays in the energy region of 5 EeV with the AMIGA upgrade of the Pierre Auger Observatory

4) G. Cowan, Statistical Data Analysis

5) Reconstruction of air shower muon densities using segmented counters with time resolution https://doi.org/10.1016/j.astropartphys.2016.06.001